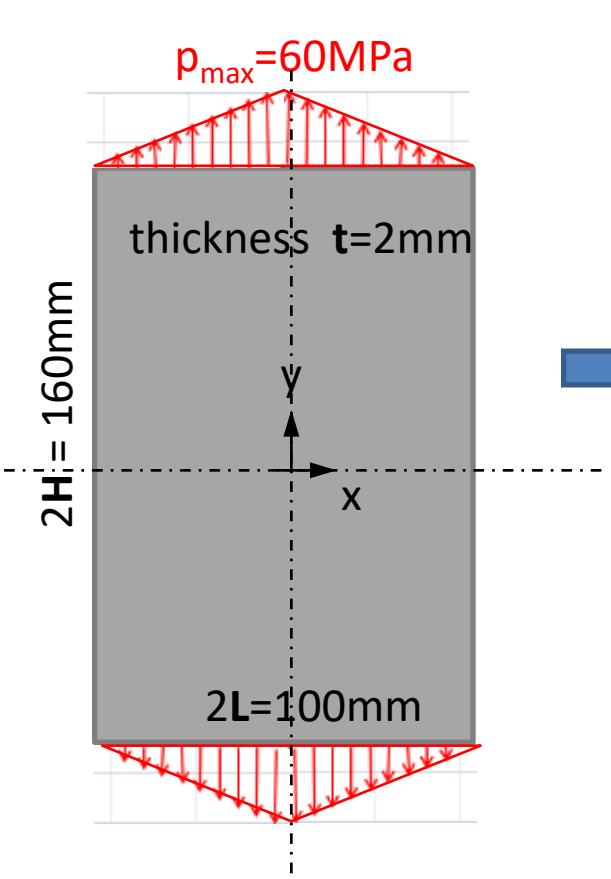
Division of Strength of Materials and Structures
Faculty of Power and Aeronautical Engineering

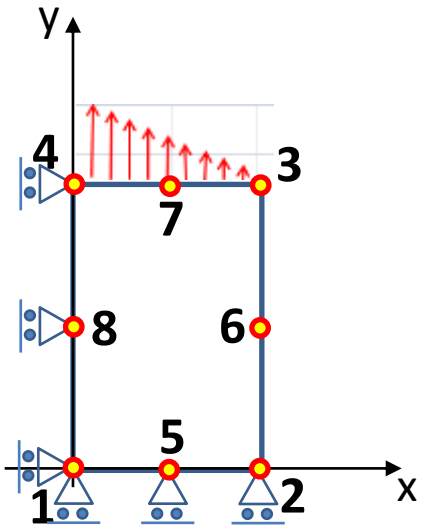
Finite element method (FEM1)

Lecture 5C. 2D Plate modeled with 8-node elements

03.2025

Example. 2D plate modeled using 8-node element





one-quarter model with single 8-node element

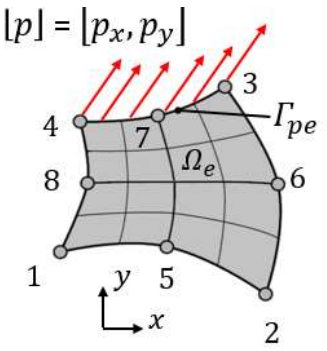
vectors of nodal coordinates:

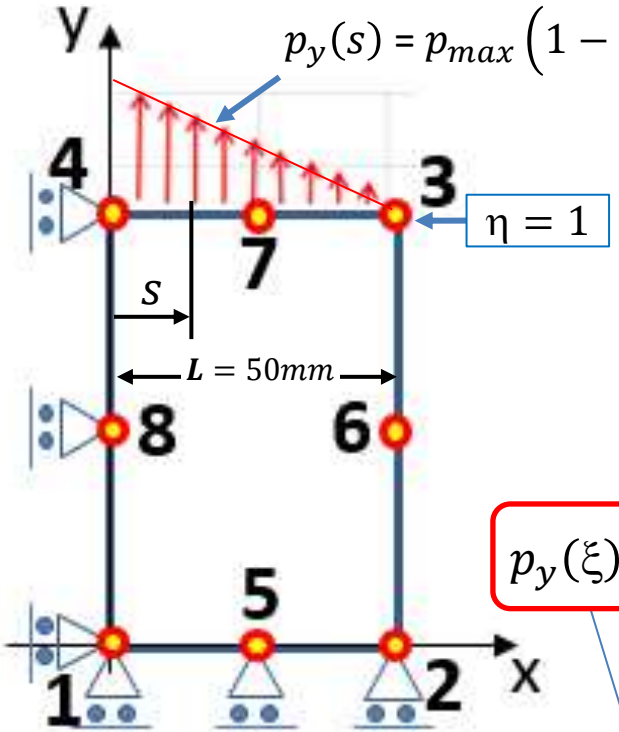
$$\lfloor x_i \rfloor_1 = \lfloor 0, L, L, 0, ½L, L, ½L, 0 \rfloor$$

$$\lfloor y_i \rfloor_1 = \lfloor 0, 0, H, H, 0, ½H, H, ½H \rfloor$$

Equivalent load vector due to surface forces

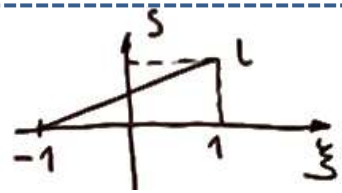
$$\underset{1\times 16}{\lfloor F^p \rfloor_e} = t_e \int_{-1}^{1} \underset{2\times 16}{\lfloor p_x, p_y \rfloor} \underset{2\times 16}{[N]} \sqrt{\left(\underset{1\times 8}{\frac{\partial \lfloor N(\xi,1) \rfloor}{\partial \xi}} \underset{8\times 1}{\{x_i\}_e}\right)^2 + \left(\underset{1\times 8}{\frac{\partial \lfloor N(\xi,1) \rfloor}{\partial \xi}} \underset{8\times 1}{\{y_i\}_e}\right)^2}\, d\xi$$





$$x_3 = L\ ,\ x_4 = 0\ \ ,\ x_7 = \frac{L}{2}$$
$$y_3 = H\ ,\ y_4 = H\ \ ,\ y_7 = H$$

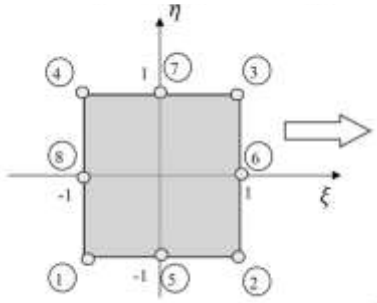
$$\left.\begin{array}{l} s{=}0 \ \rightarrow\ \xi = -1 \\ s{=}L \ \rightarrow\ \xi = +1 \end{array}\right\} \rightarrow\ s(\xi) = \frac{L}{2}(\xi + 1)$$



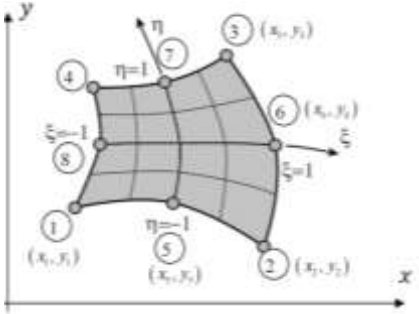
$$p_y(\xi) = p_{max}\left(1 - \frac{s(\xi)}{L}\right) = p_{max}\left(1 - \frac{1}{2}(\xi + 1)\right) = \frac{p_{max}}{2}(1 - \xi)$$

$$\underset{1\times 16}{\lfloor F^p \rfloor_1} = t \int_{-1}^{1} \underset{2\times 16}{\lfloor 0, p_y(\xi) \rfloor} \underset{2\times 16}{[N]} \sqrt{\left(\underset{1\times 8}{\frac{\partial \lfloor N(\xi,1) \rfloor}{\partial \xi}} \underset{8\times 1}{\{x_i\}_e}\right)^2 + \left(\underset{1\times 8}{\frac{\partial \lfloor N(\xi,1) \rfloor}{\partial \xi}} \underset{8\times 1}{\{y_i\}_e}\right)^2}\, d\xi$$

natural coordinate system



cartesien coordinate system



shape functions of the element and their derivatives in the natural system:

$$N_1(\xi,\eta)=-\frac{1}{4}(1-\xi)(1-\eta)(1+\xi+\eta)$$
$$N_2(\xi,\eta)=-\frac{1}{4}(1+\xi)(1-\eta)(1-\xi+\eta)$$
$$N_3(\xi,\eta)=-\frac{1}{4}(1+\xi)(1+\eta)(1-\xi-\eta)$$
$$N_4(\xi,\eta)=-\frac{1}{4}(1-\xi)(1+\eta)(1+\xi-\eta)$$
$$N_5(\xi,\eta)=\frac{1}{2}(1-\xi^2)(1-\eta)$$
$$N_6(\xi,\eta)=\frac{1}{2}(1+\xi)(1-\eta^2)$$
$$N_7(\xi,\eta)=\frac{1}{2}(1-\xi^2)(1+\eta)$$
$$N_8(\xi,\eta)=\frac{1}{2}(1-\xi)(1-\eta^2)$$

i	$N_i(\xi,1)$
1	0
2	0
3	$½(1+\xi)\xi$
4	$-½(1-\xi)\xi$
5	0
6	0
7	$1-\xi^2$
8	0

i	$\frac{\partial N_i}{\partial \xi}$	$\frac{\partial N_i}{\partial \eta}$
1	$¼(1-\eta)(2\xi+\eta)$	$¼(1-\xi)(\xi+2\eta)$
2	$¼(1-\eta)(2\xi-\eta)$	$¼(1+\xi)(2\eta-\xi)$
3	$¼(1+\eta)(2\xi+\eta)$	$¼(1+\xi)(\xi+2\eta)$
4	$¼(1+\eta)(2\xi-\eta)$	$¼(1-\xi)(2\eta-\xi)$
5	$-(1-\eta)\xi$	$-½(1-\xi^2)$
6	$½(1-\eta^2)$	$-(1+\xi)\,\eta$
7	$-(1+\eta)\,\xi$	$½(1-\xi^2)$
8	$½(1-\eta^2)$	$-(1-\xi)$

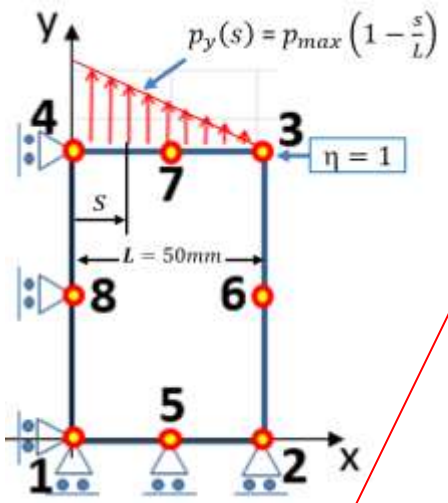
$\frac{\partial N_i(\xi,1)}{\partial \xi}$
$¼(1-1)(2\xi+1)=\mathbf{0}$
$¼(1-1)(2\xi-1)=\mathbf{0}$
$¼(1+1)(2\xi+1)=\mathbf{½(2\xi+1)}$
$¼(1+1)(2\xi-1)=\mathbf{½(2\xi-1)}$
$-(1-1)\xi=0$
$½(1-1^2)=0$
$-(1+1)\,\xi=\mathbf{-2\,\xi}$
$½(1-1^2)=0$

$x_3 = L\ ,\ x_4 = 0\ ,\ x_7 = \frac{L}{2}$

$y_3 = H\ ,\ y_4 = H\ ,\ y_7 = H$

$$\frac{\partial N_3(\xi,1)}{\partial \xi} = \tfrac{1}{2}(2\xi + 1)\ ,\quad \frac{\partial N_4(\xi,1)}{\partial \xi} = \tfrac{1}{2}(2\xi - 1)\ ,\quad \frac{\partial N_7(\xi,1)}{\partial \xi} = -2\,\xi$$

$$N_3(\xi,1) = \tfrac{1}{2}(1+\xi)\xi\ ,\quad N_4(\xi,1) = -\tfrac{1}{2}(1-\xi)\xi\ ,\quad N_7(\xi,1) = 1-\xi^2$$



$$\lfloor F^p \rfloor_1 = t \int_{-1}^{1} \lfloor 0, p_y(\xi) \rfloor [N] \sqrt{\left(\frac{\partial \lfloor N(\xi,1) \rfloor}{\partial \xi}\{x_i\}_e\right)^2 + \left(\frac{\partial \lfloor N(\xi,1) \rfloor}{\partial \xi}\{y_i\}_e\right)^2}\, d\xi$$

$$\frac{\partial \lfloor N(\xi,1) \rfloor}{\partial \xi}\{x_i\}_1 = \frac{\partial N_3(\xi,1)}{\partial \xi}x_3 + \frac{\partial N_4(\xi,1)}{\partial \xi}x_4 + \frac{\partial N_7(\xi,1)}{\partial \xi}x_7 =$$

$$= \tfrac{1}{2}(2\xi + 1)\cdot L + \tfrac{1}{2}(2\xi - 1)\cdot 0 - 2\,\xi \cdot \tfrac{L}{2} = \frac{L}{2}$$

$$\frac{\partial \lfloor N(\xi,1) \rfloor}{\partial \xi}\{y_i\}_1 = \frac{\partial N_3(\xi,1)}{\partial \xi}y_3 + \frac{\partial N_4(\xi,1)}{\partial \xi}y_4 + \frac{\partial N_7(\xi,1)}{\partial \xi}y_7 =$$

$$= \tfrac{1}{2}(2\xi + 1)\cdot H + \tfrac{1}{2}(2\xi - 1)\cdot \mathrm{H} - 2\,\xi \cdot H = 0$$

$$\sqrt{\left(\frac{\partial \lfloor N(\xi,1) \rfloor}{\partial \xi}\{x_i\}_e\right)^2 + \left(\frac{\partial \lfloor N(\xi,1) \rfloor}{\partial \xi}\{y_i\}_e\right)^2} = \frac{L}{2}$$

$$\lfloor F^p \rfloor_1 = t\frac{L}{2}\int_{-1}^{1} \lfloor 0, p_y(\xi) \rfloor [N(\xi,1)]\, d\xi$$

$$[N(\xi,1)] = \begin{bmatrix} 0 & 0 & 0 & 0 & N_3(\xi,1) & 0 & N_4(\xi,1) & 0 & 0 & 0 & 0 & 0 & N_7(\xi,1) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & N_3(\xi,1) & 0 & N_4(\xi,1) & 0 & 0 & 0 & 0 & 0 & N_7(\xi,1) & 0 & 0 \end{bmatrix}$$

$$N_3(\xi,1)= \tfrac{1}{2}(1+\xi)\xi \quad , \quad N_4(\xi,1)= -\tfrac{1}{2}(1-\xi)\xi \quad , \quad N_7(\xi,1)= 1-\xi^2$$

$$[N(\xi,1)] = \begin{bmatrix} 0 & 0 & 0 & 0 & N_3(\xi,1) & 0 & N_4(\xi,1) & 0 & 0 & 0 & 0 & 0 & N_7(\xi,1) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & N_3(\xi,1) & 0 & N_4(\xi,1) & 0 & 0 & 0 & 0 & 0 & N_7(\xi,1) & 0 & 0 \end{bmatrix}$$

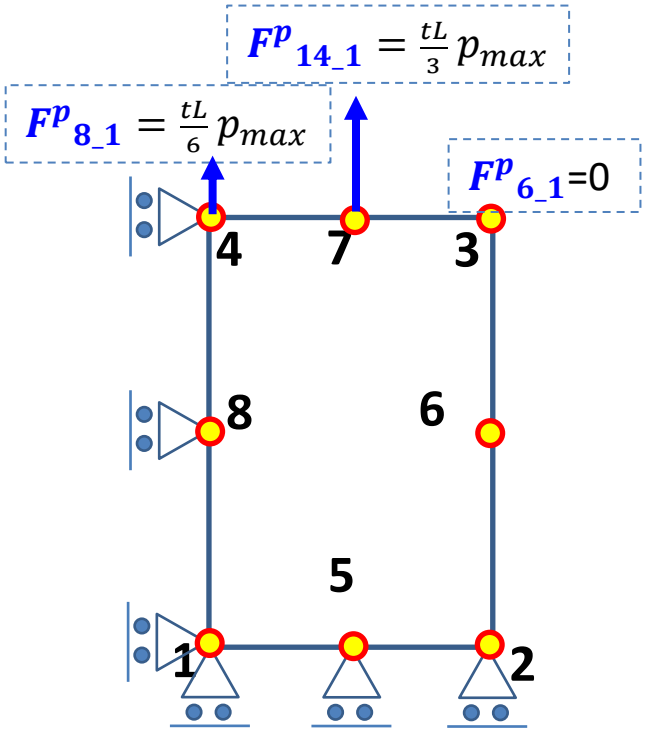
$$[F^p]_1 = t\frac{L}{2}\int_{-1}^{1}[0, p_y(\xi)][N(\xi,1)]\, d\xi$$

$$p_y(\xi) = \frac{p_{max}}{2}(1-\xi)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & N_3(\xi,1) & 0 & N_4(\xi,1) & 0 & 0 & 0 & 0 & 0 & N_7(\xi,1) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & N_3(\xi,1) & 0 & N_4(\xi,1) & 0 & 0 & 0 & 0 & 0 & N_7(\xi,1) & 0 & 0 \end{bmatrix}$$

$[0, p_y(\xi)]$ $[0,0,0,0,0, N_3(\xi,1)p_y(\xi), 0, N_4(\xi,1)p_y(\xi), 0,0,0,0,0, N_7(\xi,1)p_y(\xi), 0, 0]$

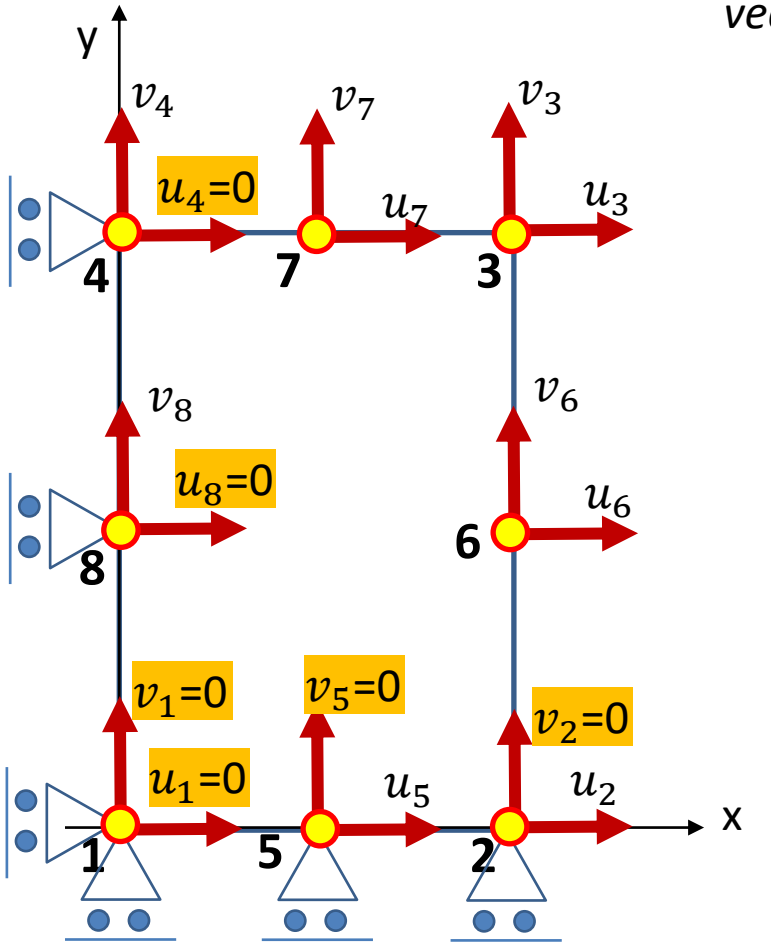
$F^p{}_{6\_1}$ $F^p{}_{8\_1}$ $F^p{}_{14\_1}$



$$F^p{}_{6\_1} = t\frac{L}{2}\int_{-1}^{1}\frac{p_{max}}{2}(1-\xi)\,\tfrac{1}{2}(1+\xi)\xi\, d\xi = 0$$

$$F^p{}_{8\_1} = t\frac{L}{2}\int_{-1}^{1}\frac{p_{max}}{2}(1-\xi)(-\tfrac{1}{2}(1-\xi)\xi)\, d\xi = \frac{tL}{6}p_{max}$$

$$F^p{}_{14\_1} = t\frac{L}{2}\int_{-1}^{1}\frac{p_{max}}{2}(1-\xi)(1-\xi^2)\, d\xi = \frac{tL}{3}p_{max}$$



vector of element nodal parameters

$$\underset{16\times 1}{\{q\}_1} = \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ \vdots \\ \vdots \\ u_8 \\ v_8 \end{Bmatrix}_1 \qquad \textit{column vector}$$

$$\underset{1\times 16}{\lfloor q \rfloor_1} = \lfloor u_1, v_1, u_2, v_2 \ldots,, u_8, v_8 \rfloor_1$$

row wector

Boundary conditions: $\mathrm{N} = 16 - 6 = 10$

vector of active nodal parameters after taking into account boundary conditions:

$$\underset{1\times 10}{\lfloor q \rfloor_1} = \lfloor u_2, u_3, v_3, v_4, u_5, u_6, v_6, u_7, v_7, v_8 \rfloor_1$$

Gradient matrix for the plain stress condition:

$$\underset{3\times 2}{[R]} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{det[J]}\frac{\partial y}{\partial \eta}\frac{\partial}{\partial \xi} - \frac{1}{det[J]}\frac{\partial y}{\partial \xi}\frac{\partial}{\partial \eta}\right) & 0 \\ 0 & \left(\frac{1}{det[J]}\frac{\partial x}{\partial \xi}\frac{\partial}{\partial \eta} - \frac{1}{det[J]}\frac{\partial x}{\partial \eta}\frac{\partial}{\partial \xi}\right) \\ \left(\frac{1}{det[J]}\frac{\partial x}{\partial \xi}\frac{\partial}{\partial \eta} - \frac{1}{det[J]}\frac{\partial x}{\partial \eta}\frac{\partial}{\partial \xi}\right) & \left(\frac{1}{det[J]}\frac{\partial y}{\partial \eta}\frac{\partial}{\partial \xi} - \frac{1}{det[J]}\frac{\partial y}{\partial \xi}\frac{\partial}{\partial \eta}\right) \end{bmatrix} =$$

$$= \begin{bmatrix} \left(\frac{1}{det[J]}\frac{\partial y}{\partial \eta}\frac{\partial}{\partial \xi}\right) & 0 \\ 0 & \left(-\frac{1}{det[J]}\frac{\partial x}{\partial \eta}\frac{\partial}{\partial \xi}\right) \\ \left(-\frac{1}{det[J]}\frac{\partial x}{\partial \eta}\frac{\partial}{\partial \xi}\right) & \left(\frac{1}{det[J]}\frac{\partial y}{\partial \eta}\frac{\partial}{\partial \xi}\right) \end{bmatrix} + \begin{bmatrix} \left(-\frac{1}{det[J]}\frac{\partial y}{\partial \xi}\frac{\partial}{\partial \eta}\right) & 0 \\ 0 & \left(\frac{1}{det[J]}\frac{\partial x}{\partial \xi}\frac{\partial}{\partial \eta}\right) \\ \left(\frac{1}{det[J]}\frac{\partial x}{\partial \xi}\frac{\partial}{\partial \eta}\right) & \left(-\frac{1}{det[J]}\frac{\partial y}{\partial \xi}\frac{\partial}{\partial \eta}\right) \end{bmatrix} =$$

$$= \begin{bmatrix} \left(\frac{1}{det[J]}\frac{\partial y}{\partial \eta}\right) & 0 \\ 0 & \left(-\frac{1}{det[J]}\frac{\partial x}{\partial \eta}\right) \\ \left(-\frac{1}{det[J]}\frac{\partial x}{\partial \eta}\right) & \left(\frac{1}{det[J]}\frac{\partial y}{\partial \eta}\right) \end{bmatrix}\frac{\partial}{\partial \xi} + \begin{bmatrix} \left(-\frac{1}{det[J]}\frac{\partial y}{\partial \xi}\right) & 0 \\ 0 & \left(\frac{1}{det[J]}\frac{\partial x}{\partial \xi}\frac{\partial}{\partial \eta}\right) \\ \left(\frac{1}{det[J]}\frac{\partial x}{\partial \xi}\right) & \left(-\frac{1}{det[J]}\frac{\partial y}{\partial \xi}\right) \end{bmatrix}\frac{\partial}{\partial \eta}$$

strain–displacement matrix

$$\underset{3\times 16}{[B]} = \underset{3\times 2}{[R]}\underset{2\times 16}{[N]} = \begin{bmatrix} \left(\frac{1}{det[J]}\frac{\partial y}{\partial \eta}\right) & 0 \\ 0 & \left(-\frac{1}{det[J]}\frac{\partial x}{\partial \eta}\right) \\ \left(-\frac{1}{det[J]}\frac{\partial x}{\partial \eta}\right) & \left(\frac{1}{det[J]}\frac{\partial y}{\partial \eta}\right) \end{bmatrix}\left[\frac{\partial N}{\partial \xi}\right] + \begin{bmatrix} \left(-\frac{1}{det[J]}\frac{\partial y}{\partial \xi}\right) & 0 \\ 0 & \left(\frac{1}{det[J]}\frac{\partial x}{\partial \xi}\frac{\partial}{\partial \eta}\right) \\ \left(\frac{1}{det[J]}\frac{\partial x}{\partial \xi}\right) & \left(-\frac{1}{det[J]}\frac{\partial y}{\partial \xi}\right) \end{bmatrix}\left[\frac{\partial N}{\partial \eta}\right]$$

$$det[J] = det\begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} = \frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \eta} - \frac{\partial y}{\partial \xi}\frac{\partial x}{\partial \eta} =$$

$$= \frac{\partial(\lfloor N(\xi,\eta)\rfloor\{x_i\}_e)}{\partial \xi}\frac{\partial(\lfloor N(\xi,\eta)\rfloor\{y_i\}_e)}{\partial \eta} - \frac{\partial(\lfloor N(\xi,\eta)\rfloor\{y_i\}_e)}{\partial \xi}\frac{\partial(\lfloor N(\xi,\eta)\rfloor\{x_i\}_e)}{\partial \eta} =$$

$$= \underset{1\times 8}{\left(\frac{\partial\lfloor N(\xi,\eta)\rfloor}{\partial \xi}\right)}\underset{8\times 1}{\{x_i\}_e}\underset{1\times 8}{\left(\frac{\partial\lfloor N(\xi,\eta)\rfloor}{\partial \eta}\right)}\underset{8\times 1}{\{y_i\}_e} - \underset{1\times 8}{\left(\frac{\partial\lfloor N(\xi,\eta)\rfloor}{\partial \xi}\right)}\underset{8\times 1}{\{y_i\}_e}\underset{1\times 8}{\left(\frac{\partial\lfloor N(\xi,\eta)\rfloor}{\partial \eta}\right)}\underset{8\times 1}{\{x_i\}_e}$$

$$\frac{\partial \xi}{\partial x} = \frac{1}{det[J]}\frac{\partial y}{\partial \eta} = \frac{1}{det[J]}\frac{\partial(\lfloor N(\xi,\eta)\rfloor\{y_i\}_e)}{\partial \eta} = \frac{1}{det[J]}\underset{1\times 8}{\frac{\partial\lfloor N(\xi,\eta)\rfloor}{\partial \eta}}\underset{8\times 1}{\{y_i\}_e}$$

$$\frac{\partial \eta}{\partial x} = -\frac{1}{det[J]}\frac{\partial y}{\partial \xi} = -\frac{1}{det[J]}\frac{\partial(\lfloor N(\xi,\eta)\rfloor\{y_i\}_e)}{\partial \xi} = -\frac{1}{det[J]}\underset{1\times 8}{\frac{\partial\lfloor N(\xi,\eta)\rfloor}{\partial \xi}}\underset{8\times 1}{\{y_i\}_e}$$

$$\frac{\partial \xi}{\partial y} = -\frac{1}{det[J]}\frac{\partial x}{\partial \eta} = -\frac{1}{det[J]}\frac{\partial(\lfloor N(\xi,\eta)\rfloor\{x_i\}_e)}{\partial \eta} = -\frac{1}{det[J]}\underset{1\times 8}{\frac{\partial\lfloor N(\xi,\eta)\rfloor}{\partial \eta}}\underset{8\times 1}{\{x_i\}_e}$$

$$\frac{\partial \eta}{\partial y} = \frac{1}{det[J]}\frac{\partial x}{\partial \xi} = \frac{1}{det[J]}\frac{\partial(\lfloor N(\xi,\eta)\rfloor\{x_i\}_e)}{\partial \xi} = \frac{1}{det[J]}\underset{1\times 8}{\frac{\partial\lfloor N(\xi,\eta)\rfloor}{\partial \xi}}\underset{8\times 1}{\{x_i\}_e}$$

$$\underset{2\times 16}{\left[\frac{\partial N}{\partial \xi}\right]} = \begin{bmatrix} \frac{\partial N_1}{\partial \xi} & 0 & \frac{\partial N_2}{\partial \xi} & 0 & \frac{\partial N_3}{\partial \xi} & 0 & \frac{\partial N_4}{\partial \xi} & 0 & \frac{\partial N_5}{\partial \xi} & 0 & \frac{\partial N_6}{\partial \xi} & 0 & \frac{\partial N_7}{\partial \xi} & 0 & \frac{\partial N_8}{\partial \xi} & 0 \\ 0 & \frac{\partial N_1}{\partial \xi} & 0 & \frac{\partial N_2}{\partial \xi} & 0 & \frac{\partial N_3}{\partial \xi} & 0 & \frac{\partial N_4}{\partial \xi} & 0 & \frac{\partial N_5}{\partial \xi} & 0 & \frac{\partial N_6}{\partial \xi} & 0 & \frac{\partial N_7}{\partial \xi} & 0 & \frac{\partial N_8}{\partial \xi} \end{bmatrix}$$

$$\underset{2\times 16}{\left[\frac{\partial N}{\partial \eta}\right]} = \begin{bmatrix} \frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \frac{\partial N_3}{\partial \eta} & 0 & \frac{\partial N_4}{\partial \eta} & 0 & \frac{\partial N_5}{\partial \eta} & 0 & \frac{\partial N_6}{\partial \eta} & 0 & \frac{\partial N_7}{\partial \eta} & 0 & \frac{\partial N_8}{\partial \eta} & 0 \\ 0 & \frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \frac{\partial N_3}{\partial \eta} & 0 & \frac{\partial N_4}{\partial \eta} & 0 & \frac{\partial N_5}{\partial \eta} & 0 & \frac{\partial N_6}{\partial \eta} & 0 & \frac{\partial N_7}{\partial \eta} & 0 & \frac{\partial N_8}{\partial \eta} \end{bmatrix}$$

$$\underset{3\times 3}{[D]} = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix}$$

constitutive matrix for plain stress

element stiffness matrix:

$$\underset{16\times 16}{[k]_e} = t_e \int_{-1}^{1}\int_{-1}^{1} \underset{16\times 3}{[B(\xi,\eta)]^T} \underset{3\times 3}{[D]} \underset{3\times 16}{[B(\xi,\eta)]} det[J]\, d\xi d\eta$$

$$\underset{16\times16}{[k]_e} = t_e \int_{-1}^{1}\int_{-1}^{1} \underset{16\times3}{[B(\xi,\eta)]^T} \underset{3\times3}{[D]} \underset{3\times16}{[B(\xi,\eta)]} det[J]\, d\xi d\eta$$

Numerical integration (2 x 2 Gauss points)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	157054.6148	48611.11111	88708.77897	259.9524658	78527.30739	20016.33987	52640.37433	-259.9524658	-218143.4442	-23915.62686	-42458.90275	-11437.9085	-114595.712	-11437.9085	-1733.016439	-21836.00713
2	48611.11111	101294.8109	-259.9524658	44529.8574	20016.33987	50647.40543	259.9524658	46635.47237	-21836.00713	-64589.52268	-11437.9085	-52753.0204	-11437.9085	-48541.79045	-23915.62686	-77223.21252
3	88708.77897	-259.9524658	157054.6148	-48611.11111	52640.37433	259.9524658	78527.30739	-20016.33987	-218143.4442	23915.62686	-1733.016439	21836.00713	-114595.712	11437.9085	-42458.90275	11437.9085
4	259.9524658	44529.8574	-48611.11111	101294.8109	-259.9524658	46635.47237	-20016.33987	50647.40543	21836.00713	-64589.52268	23915.62686	-77223.21252	11437.9085	-48541.79045	11437.9085	-52753.0204
5	78527.30739	20016.33987	52640.37433	-259.9524658	157054.6148	48611.11111	88708.77897	259.9524658	-114595.712	-11437.9085	-1733.016439	-21836.00713	-218143.4442	-23915.62686	-42458.90275	-11437.9085
6	20016.33987	50647.40543	259.9524658	46635.47237	48611.11111	101294.8109	-259.9524658	44529.8574	-11437.9085	-48541.79045	-23915.62686	-77223.21252	-21836.00713	-64589.52268	-11437.9085	-52753.0204
7	52640.37433	259.9524658	78527.30739	-20016.33987	88708.77897	-259.9524658	157054.6148	-48611.11111	-114595.712	11437.9085	-42458.90275	11437.9085	-218143.4442	23915.62686	-1733.016439	21836.00713
8	-259.9524658	46635.47237	-20016.33987	50647.40543	259.9524658	44529.8574	-48611.11111	101294.8109	11437.9085	-48541.79045	11437.9085	-52753.0204	21836.00713	-64589.52268	23915.62686	-77223.21252
9	-218143.4442	-21836.00713	-218143.4442	21836.00713	-114595.712	-11437.9085	-114595.712	11437.9085	458382.8481	-1.81899E-11	-1.81899E-11	-45751.63399	207095.4644	-9.09495E-12	5.45697E-12	45751.63399
10	-23915.62686	-64589.52268	23915.62686	-64589.52268	-11437.9085	-48541.79045	11437.9085	-48541.79045	-1.81899E-11	194167.1618	-45751.63399	7.27596E-12	-7.27596E-12	32095.46445	45751.63399	-3.63798E-12
11	-42458.90275	-11437.9085	-1733.016439	23915.62686	-1733.016439	-23915.62686	-42458.90275	11437.9085	-1.45519E-11	-45751.63399	169835.611	7.27596E-12	1.45519E-11	45751.63399	-81451.77263	5.45697E-12
12	-11437.9085	-52753.0204	21836.00713	-77223.21252	-21836.00713	-77223.21252	11437.9085	-52753.0204	-45751.63399	5.45697E-12	-7.27596E-12	211012.0816	45751.63399	-1.81899E-11	5.45697E-12	48940.38423
13	-114595.712	-11437.9085	-114595.712	11437.9085	-218143.4442	-21836.00713	-218143.4442	21836.00713	207095.4644	-9.09495E-12	0	45751.63399	458382.8481	-1.09139E-11	0	-45751.63399
14	-11437.9085	-48541.79045	11437.9085	-48541.79045	-23915.62686	-64589.52268	23915.62686	-64589.52268	-7.27596E-12	32095.46445	45751.63399	-1.09139E-11	-1.09139E-11	194167.1618	-45751.63399	0
15	-1733.016439	-23915.62686	-42458.90275	11437.9085	-42458.90275	-11437.9085	-1733.016439	23915.62686	2.00089E-11	45751.63399	-81451.77263	7.27596E-12	-2.91038E-11	-45751.63399	169835.611	0
16	-21836.00713	-77223.21252	11437.9085	-52753.0204	-11437.9085	-52753.0204	21836.00713	-77223.21252	45751.63399	-3.63798E-12	3.63798E-12	48940.38423	-45751.63399	0	1.09139E-11	211012.0816

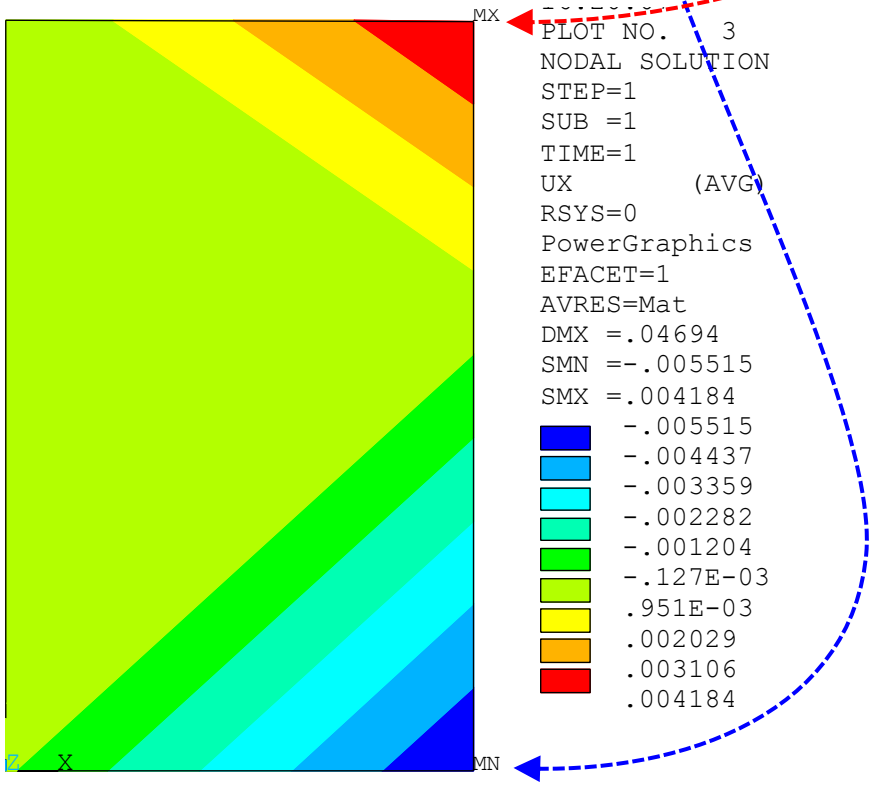
[K] NxN										
157054.6148	52640	259.9524658	-20016.33987	-2E+05	-1733	21836.00713	-1E+05	11437.9085	11438
52640.37433	157055	48611.11111	259.9524658	-1E+05	-1733	-21836.0071	-2E+05	-23915.6269	-11438
259.9524658	48611	101294.8109	44529.8574	-11438	-23916	-77223.2125	-21836	-64589.5227	-52753
-20016.33987	259.95	44529.8574	101294.8109	11438	11438	-52753.0204	21836	-64589.5227	-77223
-218143.4442	-1E+05	-11437.9085	11437.9085	458383	-2E-11	-45751.634	207095	-9.0949E-12	45752
-1733.016439	-1733	-23915.6269	11437.9085	-1E-11	169836	7.27596E-12	1E-11	45751.634	5E-12
21836.00713	-21836	-77223.2125	-52753.0204	-45752	-7E-12	211012.0816	45752	-1.819E-11	48940
-114595.712	-2E+05	-21836.0071	21836.00713	207095	0	45751.63399	458383	-1.0914E-11	-45752
11437.9085	-23916	-64589.5227	-64589.52268	-7E-12	45752	-1.0914E-11	-1E-11	194167.162	0
11437.9085	-11438	-52753.0204	-77223.21252	45752	4E-12	48940.38423	-45752	0	211012

$$\{q\} = [k]^{-1}\{F\}$$

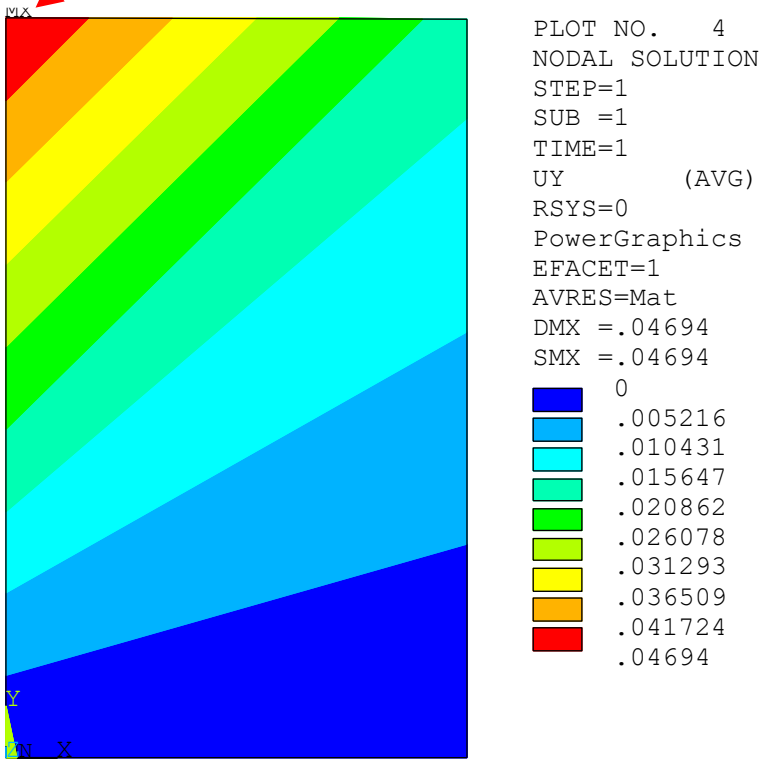
10×1 10×10 10×1

F		q	
0		-0.00523	u2
0		0.004468	u3
0		0.0181	v3
1000	N	0.046933	v4
0		-0.00353	u5
0		-0.01009	u6
0		0.013534	v6
0		0.001708	u7
2000	N	0.035171	v7
0		0.020224	v8

UX displacement

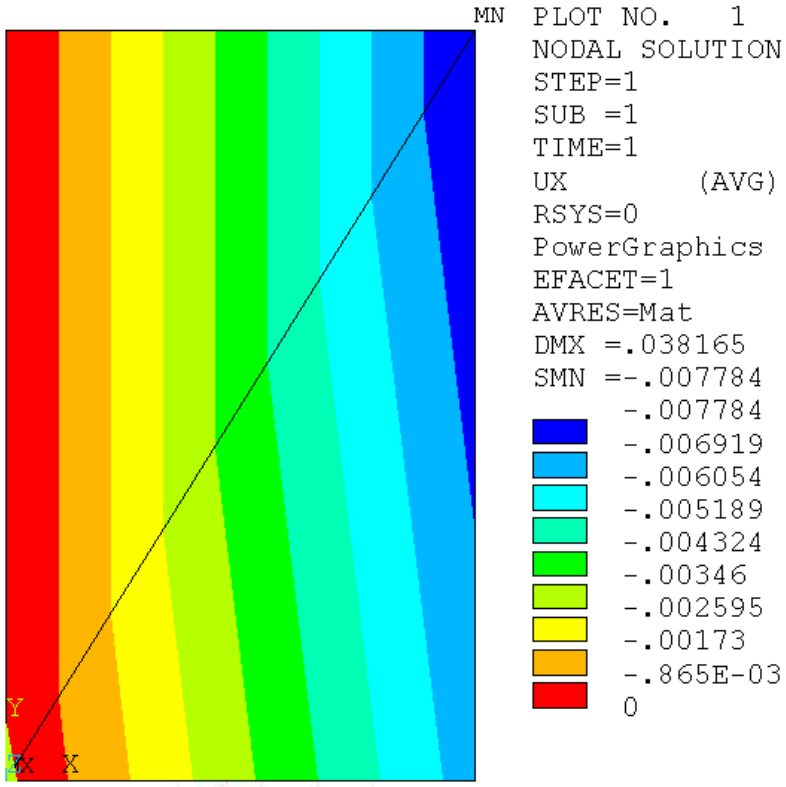


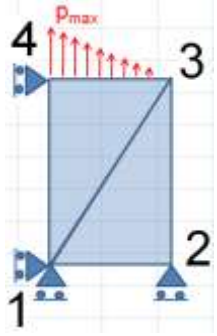
UY displacement

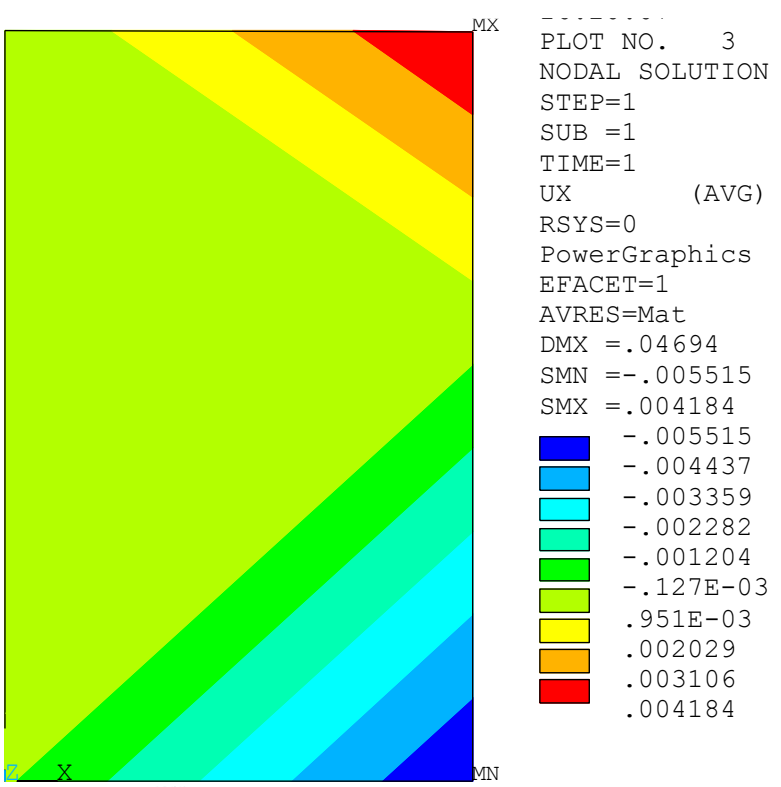


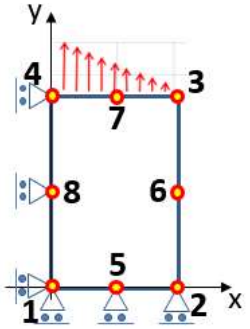
Displacements in X direction

UX displacement



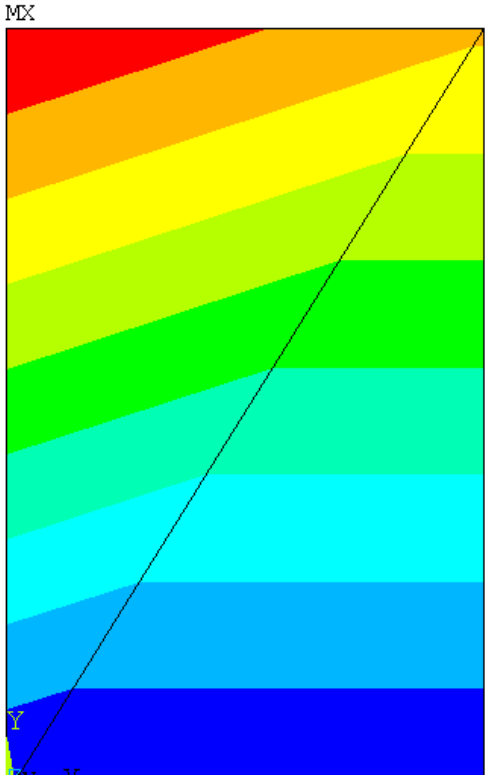






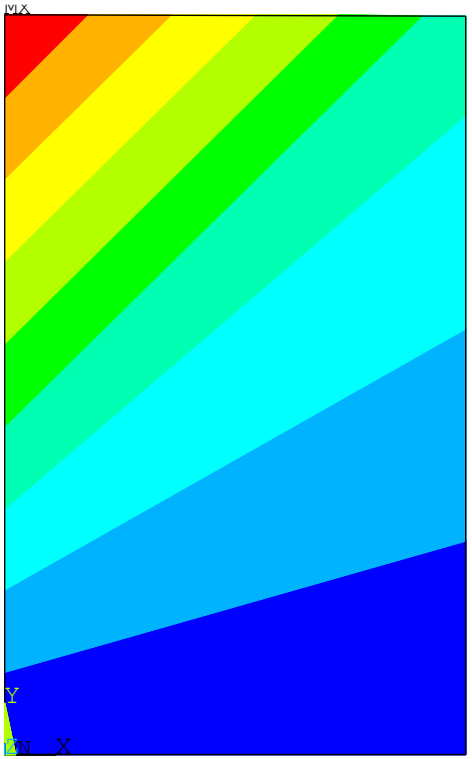
Displacements in Y direction

UY displacement



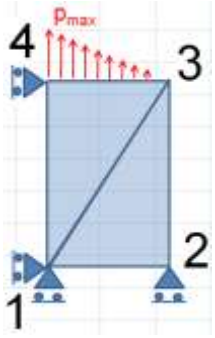
PLOT NO. 2
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.038165
SMX =.038165

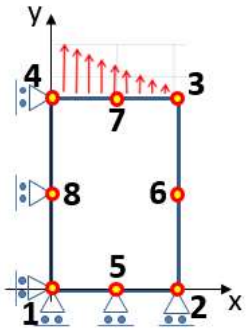
0
.004241
.008481
.012722
.016962
.021203
.025444
.029684
.033925
.038165



PLOT NO. 4
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.04694
SMX =.04694

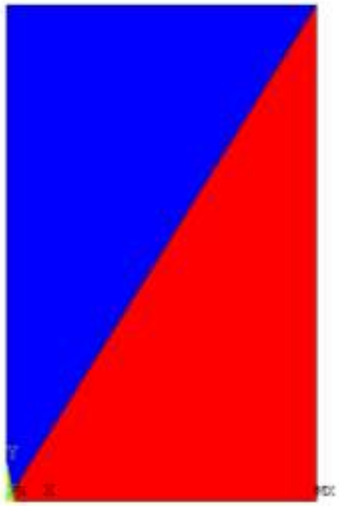
0
.005216
.010431
.015647
.020862
.026078
.031293
.036509
.041724
.04694



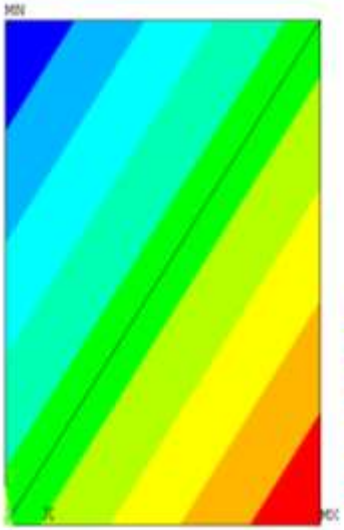


Strain in X direction

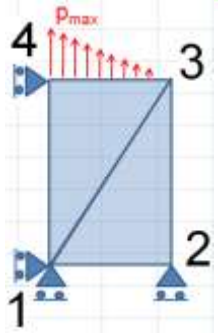
ε_x strain

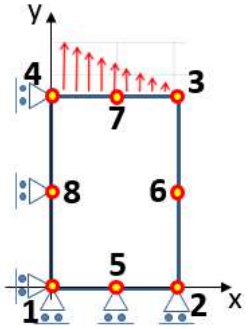


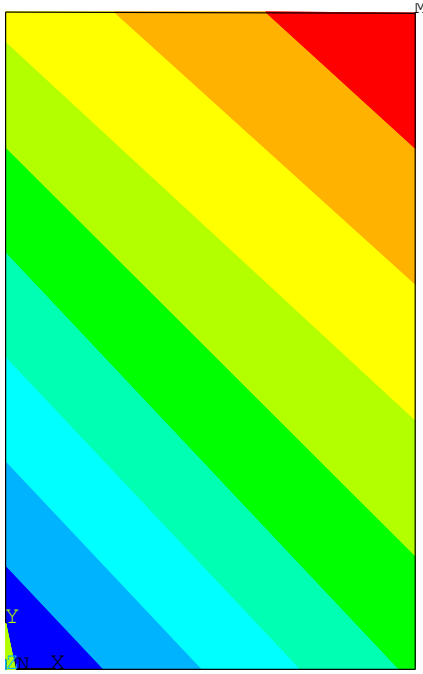
PLOT NO. 7
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
EPELX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.038165
SMN =-.156E-03
SMX =-.130E-03
-.156E-03
-.153E-03
-.150E-03
-.147E-03
-.144E-03
-.141E-03
-.139E-03
-.136E-03
-.133E-03
-.130E-03



PLOT NO. 12
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
EPELX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.038165
SMN =-.156E-03
SMX =-.130E-03
-.156E-03
-.153E-03
-.150E-03
-.147E-03
-.144E-03
-.141E-03
-.139E-03
-.136E-03
-.133E-03
-.130E-03



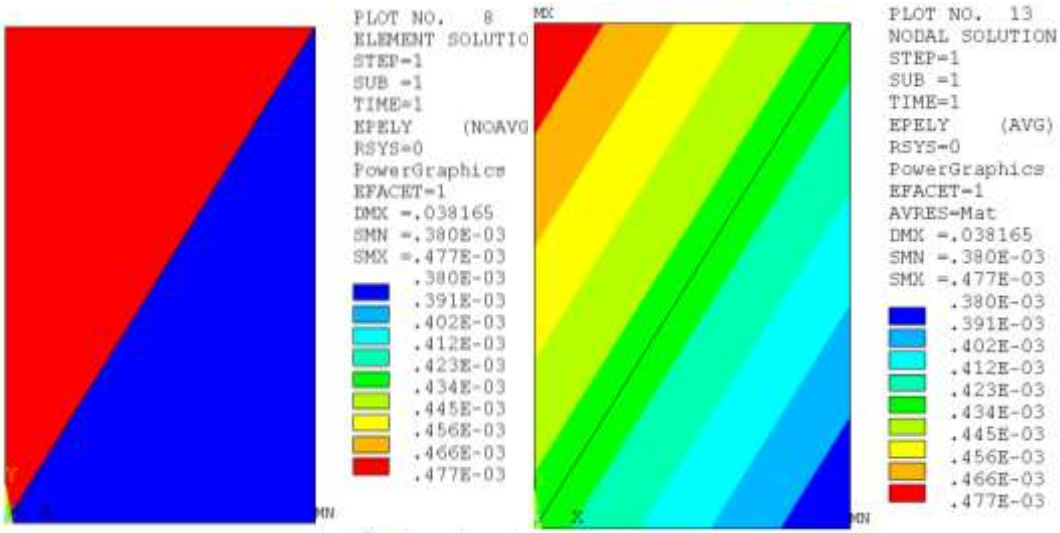


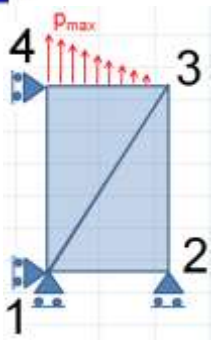


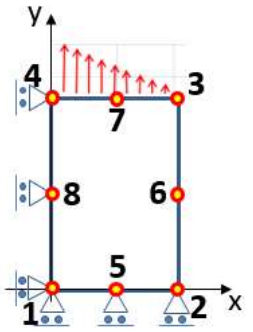
PLOT NO. 12
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
EPELX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.04694
SMN =-.313E-03
SMX =.157E-05
-.313E-03
-.278E-03
-.243E-03
-.208E-03
-.173E-03
-.138E-03
-.103E-03
-.682E-04
-.333E-04
.157E-05

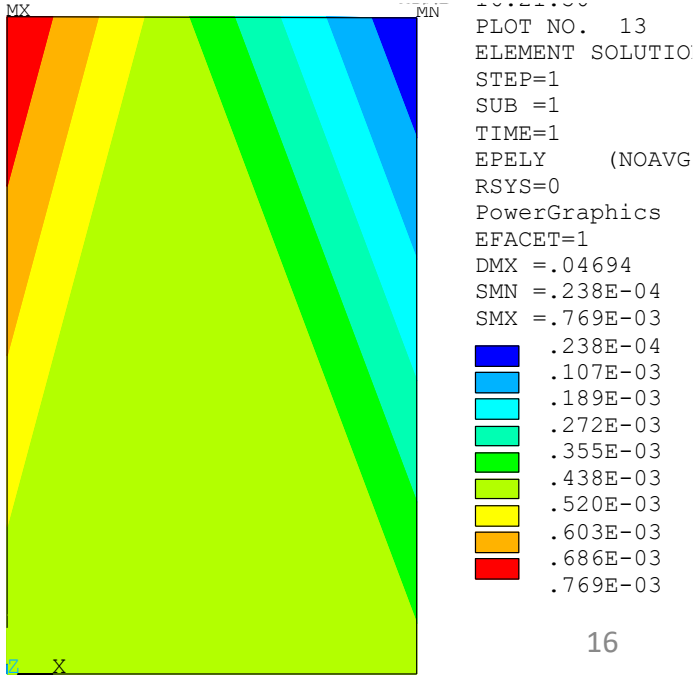
Strain in Y direction

ε_y strain



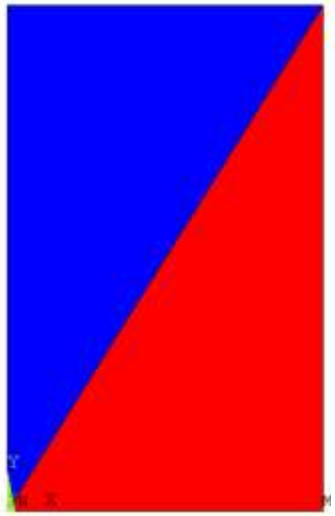




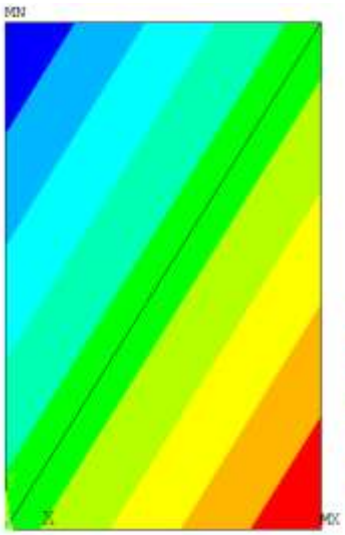


Shear strain

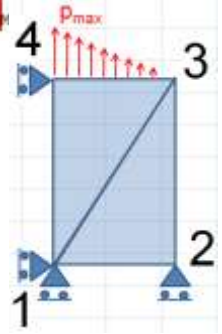
γ_{xy} strain

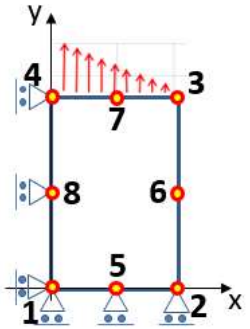


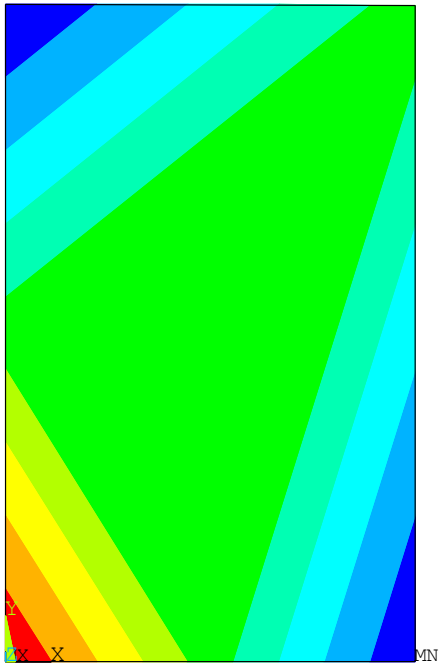
PLOT NO. 1
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
EPELXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.038165
SMN =-.155E-03
SMX =-.160E-04
-.155E-03
-.140E-03
-.124E-03
-.109E-03
-.933E-04
-.779E-04
-.624E-04
-.470E-04
-.315E-04
-.160E-04



PLOT NO. 14
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
EPELXY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.038165
SMN =-.155E-03
SMX =-.160E-04
-.155E-03
-.140E-03
-.124E-03
-.109E-03
-.933E-04
-.779E-04
-.624E-04
-.470E-04
-.315E-04
-.160E-04



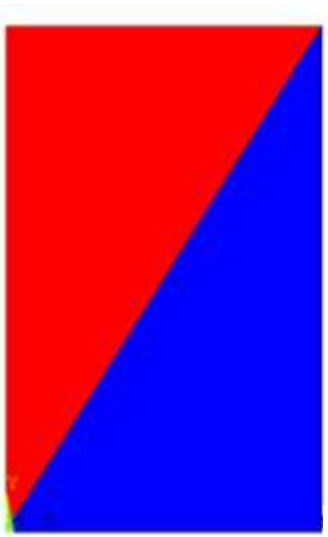


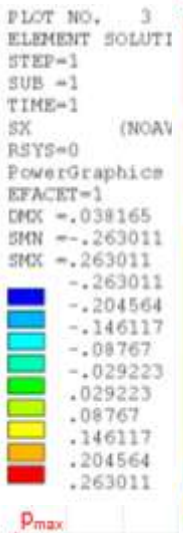


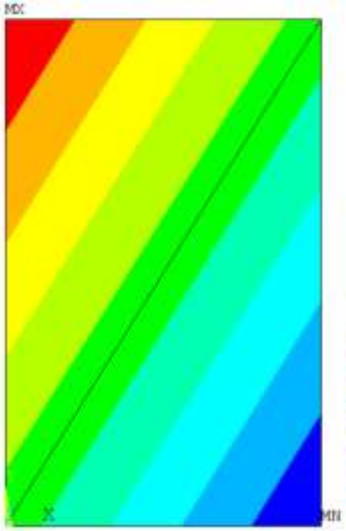
PLOT NO. 14
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
EPELXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.04694
SMN =-.259E-03
SMX =.105E-03
-.259E-03
-.219E-03
-.178E-03
-.138E-03
-.974E-04
-.569E-04
-.164E-04
.241E-04
.646E-04
.105E-03

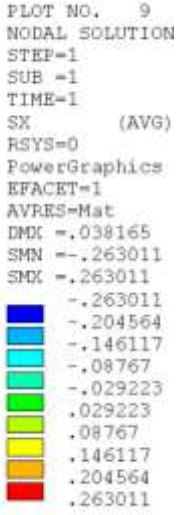
Stress in X direction

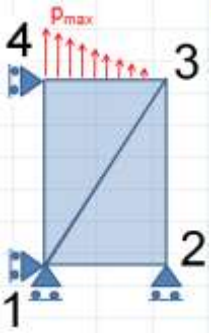
σ_x stress

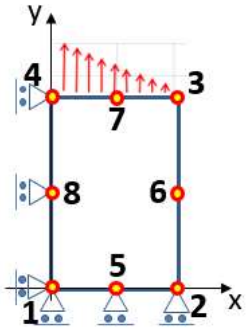


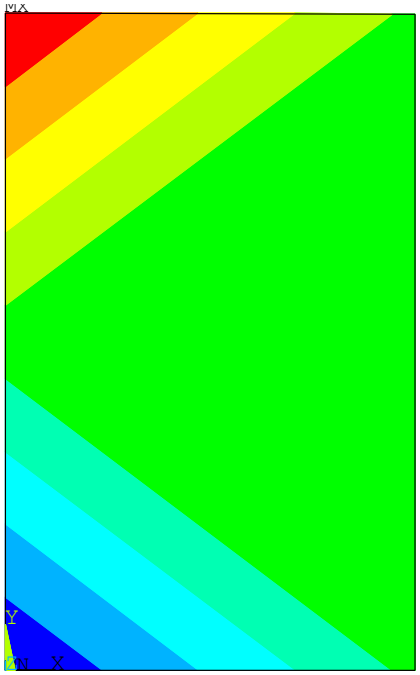


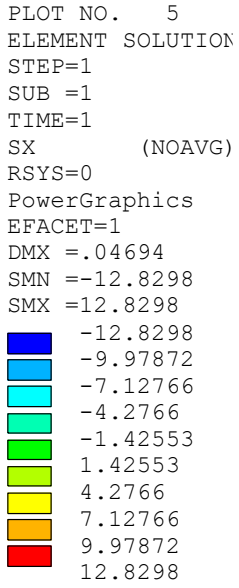






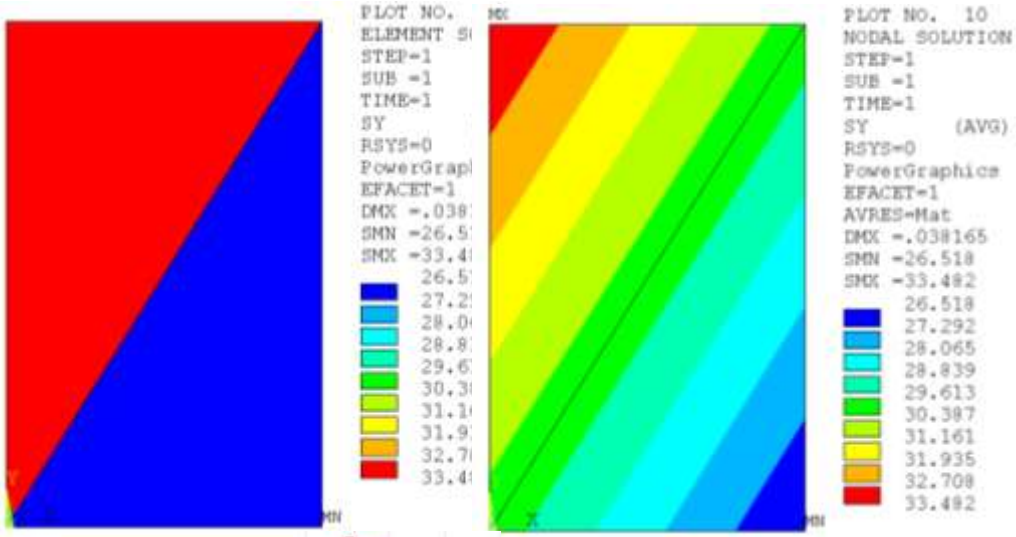


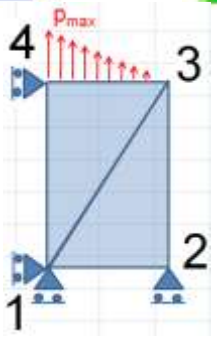


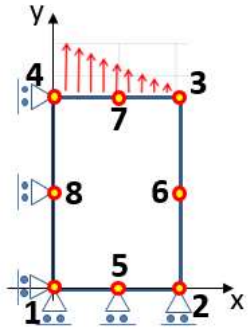


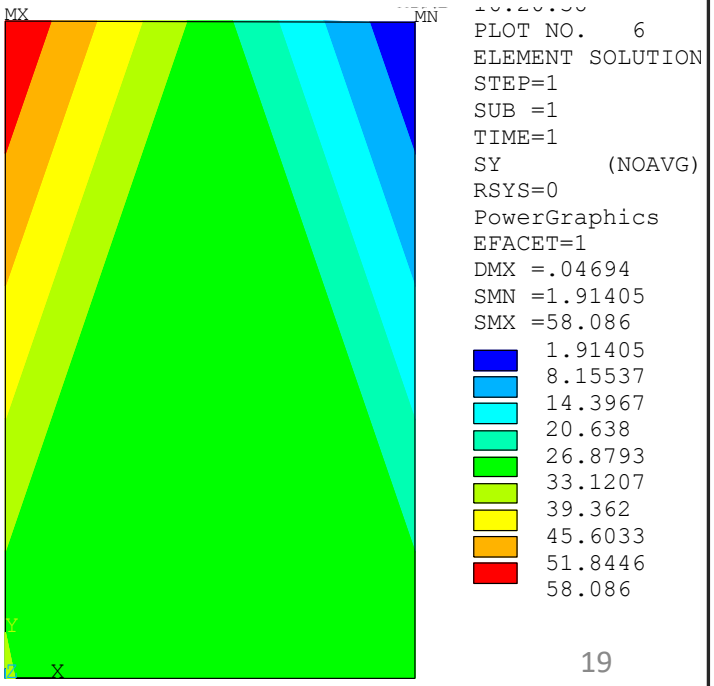
Stress in Y direction

σ_y stress



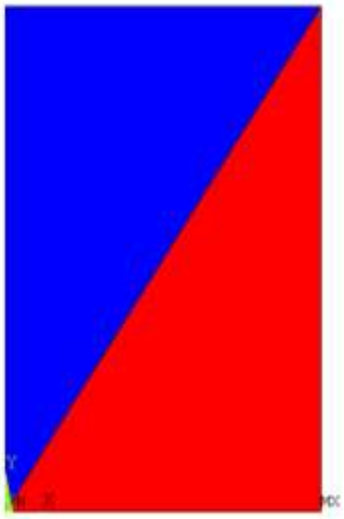


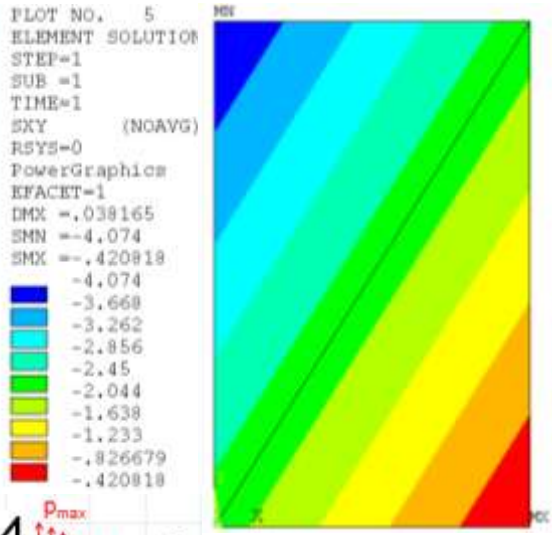


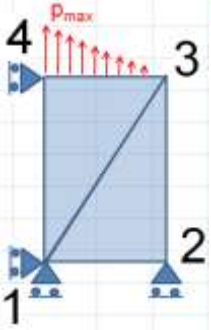


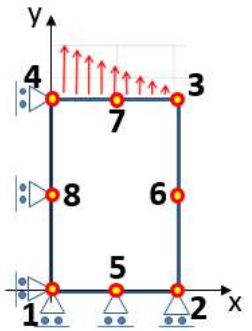
Shear stress

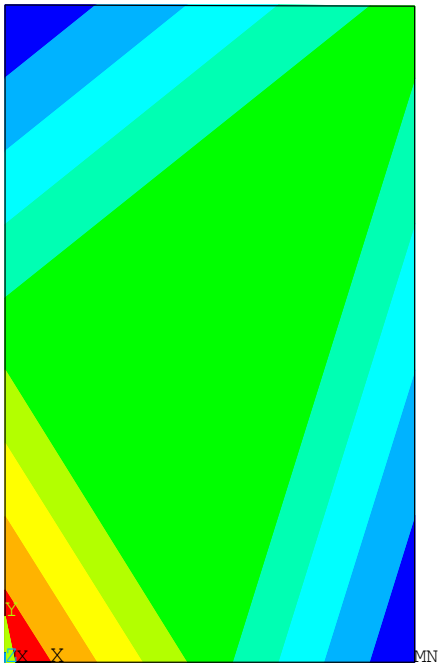
τ_{xy} stress









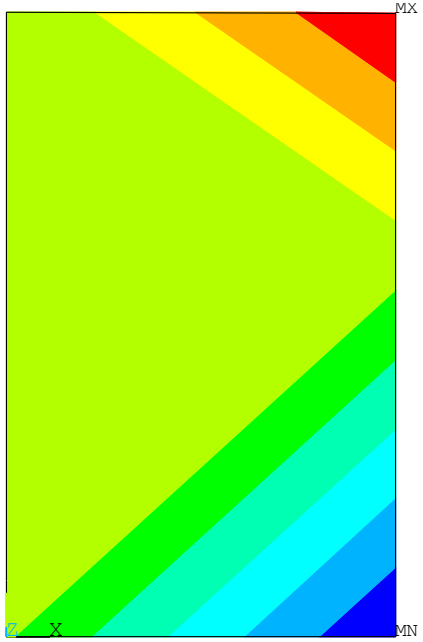


The impact of discretization on the quality of results *(8-node elements)*

UX

[mm]

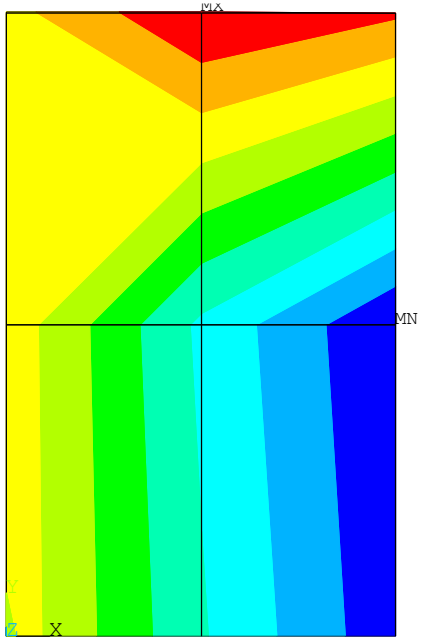
1



PLOT NO. 3
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.04694
SMN =-.005515
SMX =.004184

-.005515
-.004437
-.003359
-.002282
-.001204
-.127E-03
.951E-03
.002029
.003106
.004184

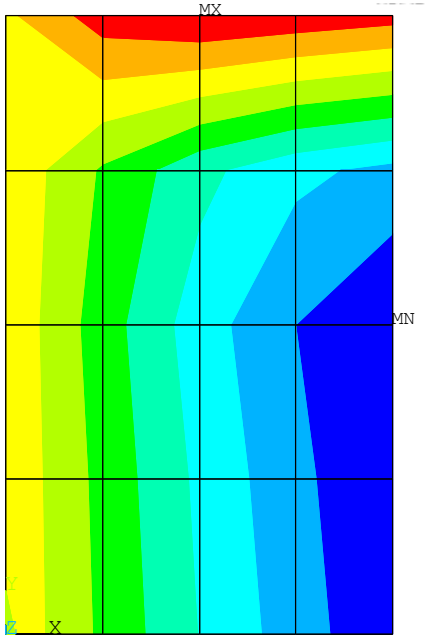
4



PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.046767
SMN =-.009058
SMX =.0032

-.009058
-.007696
-.006334
-.004972
-.00361
-.002248
-.886E-03
.476E-03
.001838
.0032

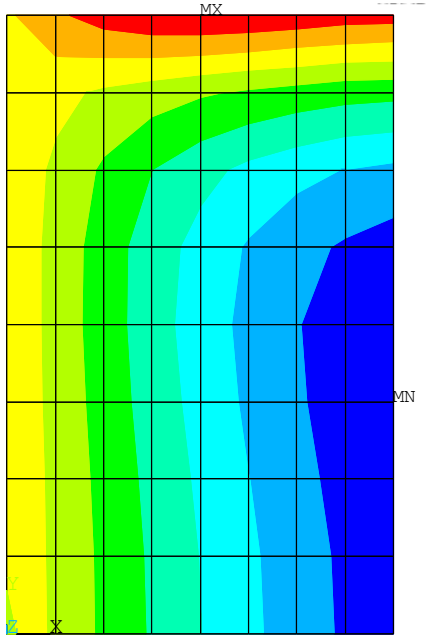
16



PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.047092
SMN =-.008831
SMX =.002854

-.008831
-.007533
-.006234
-.004936
-.003637
-.002339
-.001041
.258E-03
.001556
.002854

64



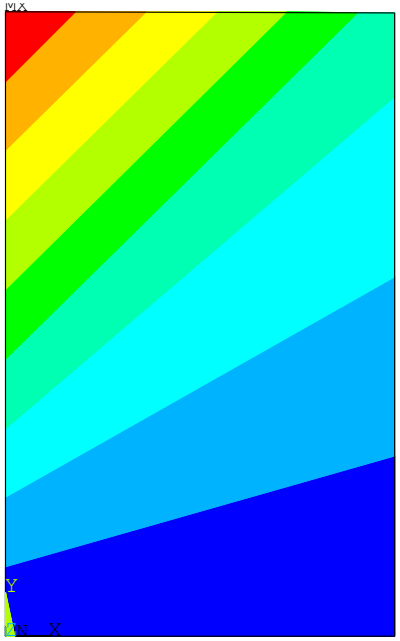
PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.047175
SMN =-.008909
SMX =.002836

-.008909
-.007604
-.006299
-.004994
-.003689
-.002384
-.001079
.226E-03
.001531
.002836

The impact of discretization on the quality of results *(8-node elements)*

UY [mm]

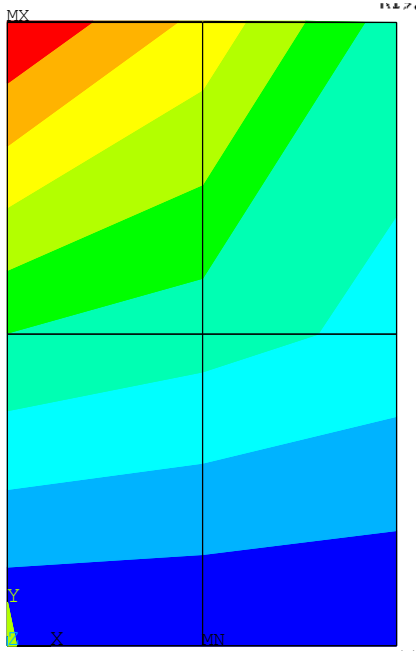
1



PLOT NO. 4
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.04694
SMX =.04694

	0
	.005216
	.010431
	.015647
	.020862
	.026078
	.031293
	.036509
	.041724
	.04694

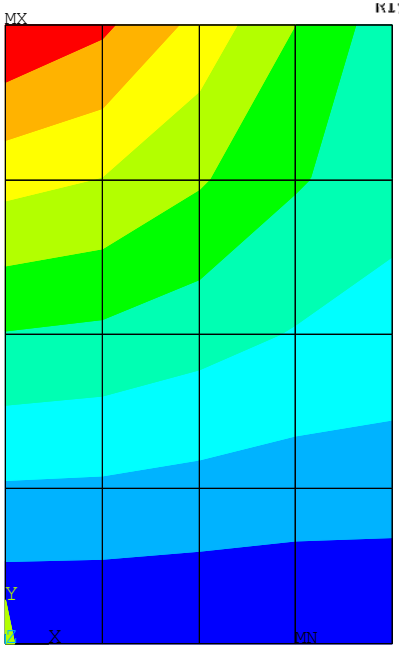
4



PLOT NO. 2
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.046767
SMX =.046767

	0
	.005196
	.010393
	.015589
	.020785
	.025982
	.031178
	.036374
	.041571
	.046767

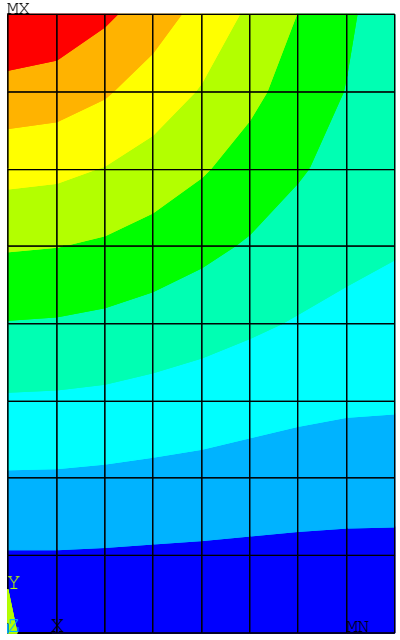
16



PLOT NO. 2
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.047092
SMX =.047092

	0
	.005232
	.010465
	.015697
	.02093
	.026162
	.031395
	.036627
	.04186
	.047092

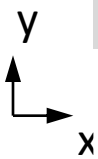
64



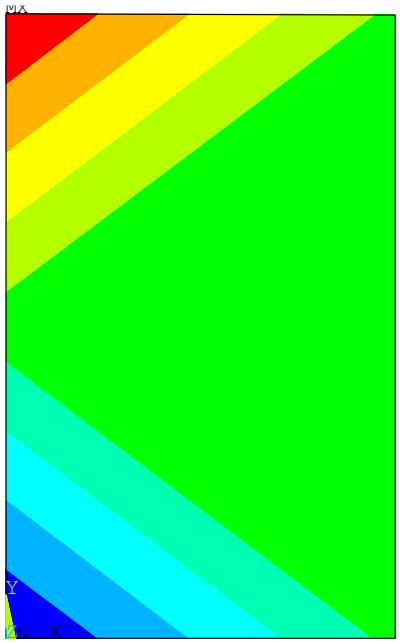
PLOT NO. 2
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.047175
SMX =.047175

	0
	.005242
	.010483
	.015725
	.020967
	.026208
	.03145
	.036692
	.041933
	.047175

The impact of discretization on the quality of results *(8-node elements)*



1



PLOT NO. 5
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.04694
SMN =-12.8298
SMX =12.8298

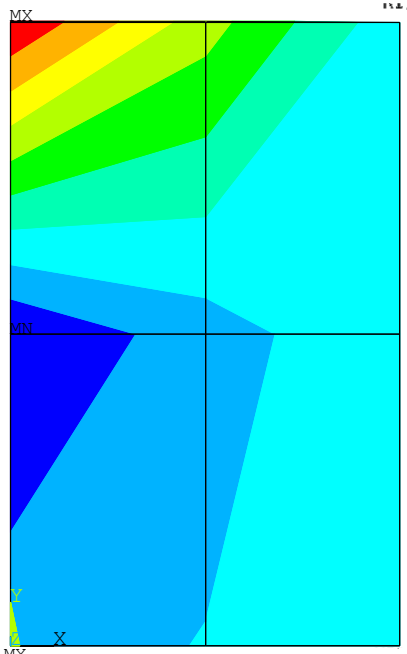
-12.8298
-9.97872
-7.12766
-4.2766
-1.42553
1.42553
4.2766
7.12766
9.97872
12.8298

σ_x

[MPa]

NODAL SOLUTION

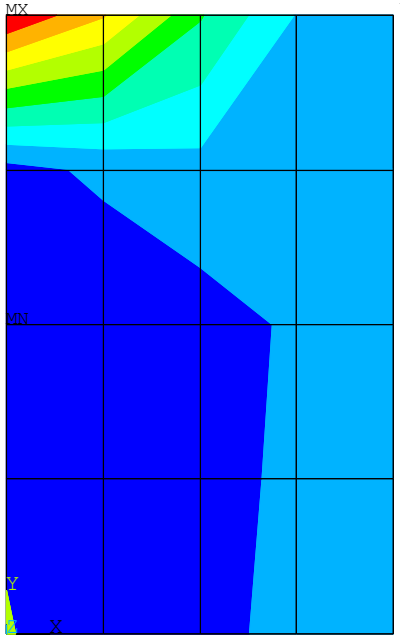
4



PLOT NO. 3
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.046767
SMN =-9.09511
SMX =21.89

-9.09511
-5.65232
-2.20952
1.23327
4.67606
8.11886
11.5617
15.0044
18.4472
21.89

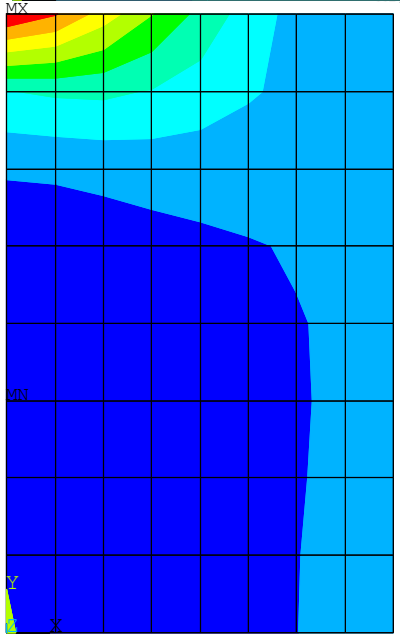
16



PLOT NO. 3
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.047092
SMN =-5.09635
SMX =28.663

-5.09635
-1.34531
2.40573
6.15677
9.90781
13.6589
17.4099
21.1609
24.912
28.663

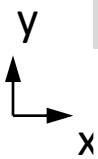
64



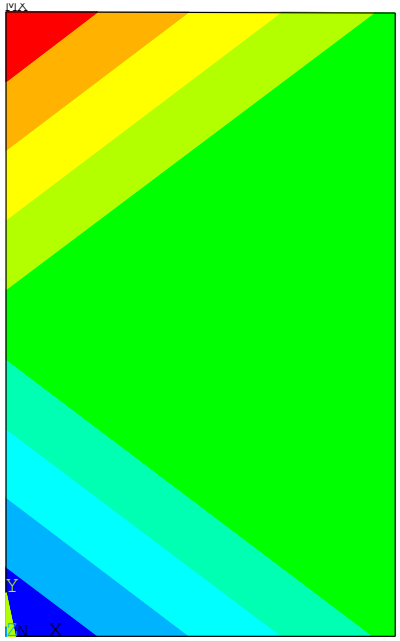
PLOT NO. 3
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.047175
SMN =-4.80315
SMX =32.5558

-4.80315
-.652148
3.49885
7.64985
11.8009
15.9518
20.1028
24.2538
28.4048
32.5558

The impact of discretization on the quality of results *(8-node elements)*



1



PLOT NO. 5
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.04694
SMN =-12.8298
SMX =12.8298

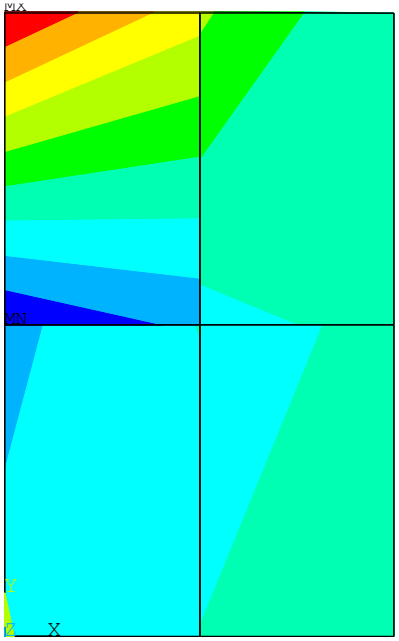
-12.8298
-9.97872
-7.12766
-4.2766
-1.42553
1.42553
4.2766
7.12766
9.97872
12.8298

σ_x

[MPa]

ELEMENT SOLUTION

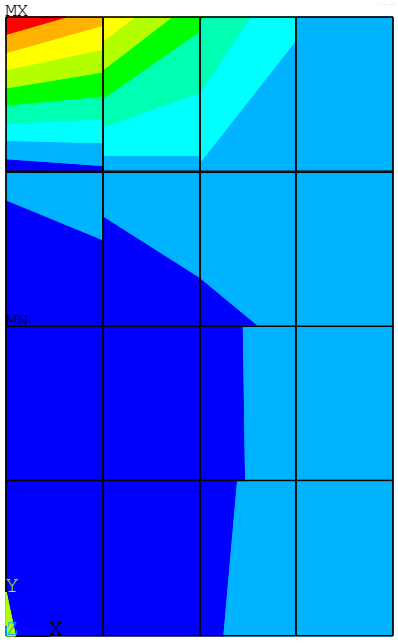
4



PLOT NO. 9
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.046767
SMN =-12.4303
SMX =21.89

-12.4303
-8.6169
-4.80353
-.990168
2.8232
6.63657
10.4499
14.2633
18.0767
21.89

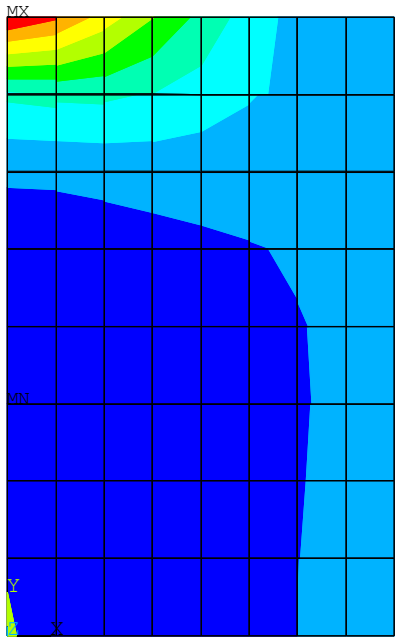
16



PLOT NO. 6
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.047092
SMN =-5.50981
SMX =28.663

-5.50981
-1.71283
2.08415
5.88113
9.67811
13.4751
17.2721
21.0691
24.866
28.663

64



PLOT NO. 6
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.047175
SMN =-4.82592
SMX =32.5558

-4.82592
-.672393
3.48114
7.63467
11.7882
15.9417
20.0953
24.2488
28.4023
32.5558

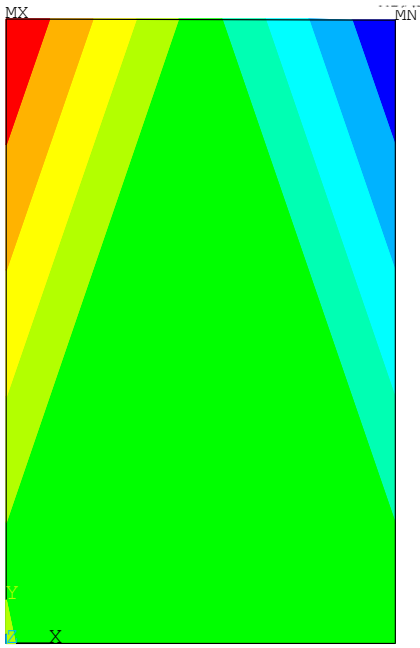
The impact of discretization on the quality of results *(8-node elements)*

σ_y

[MPa]

NODAL SOLUTION

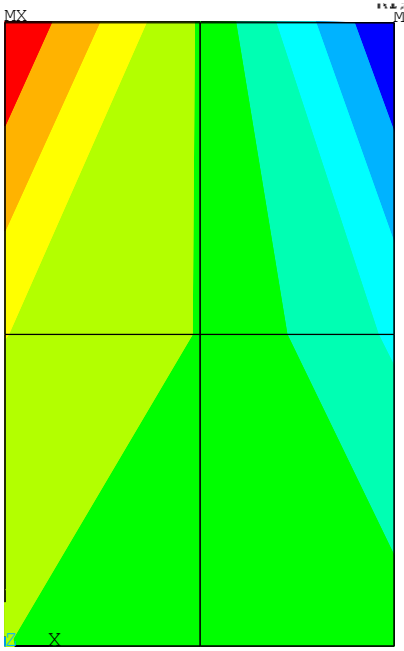
1



PLOT NO. 6
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.04694
SMN =1.91405
SMX =58.086

1.91405
8.15537
14.3967
20.638
26.8793
33.1207
39.362
45.6033
51.8446
58.086

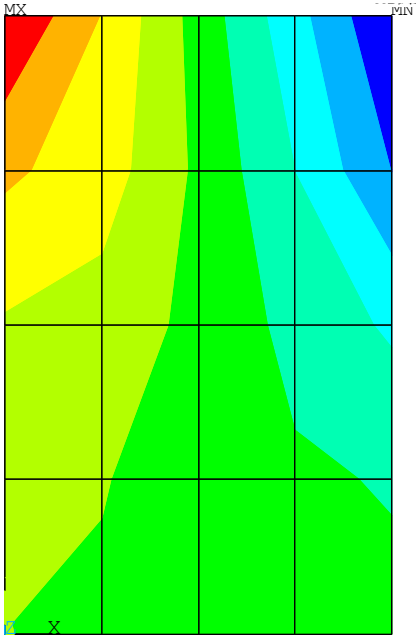
4



PLOT NO. 4
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.046767
SMN =-1.56578
SMX =58.9382

-1.56578
5.15688
11.8795
18.6022
25.3249
32.0475
38.7702
45.4928
52.2155
58.9382

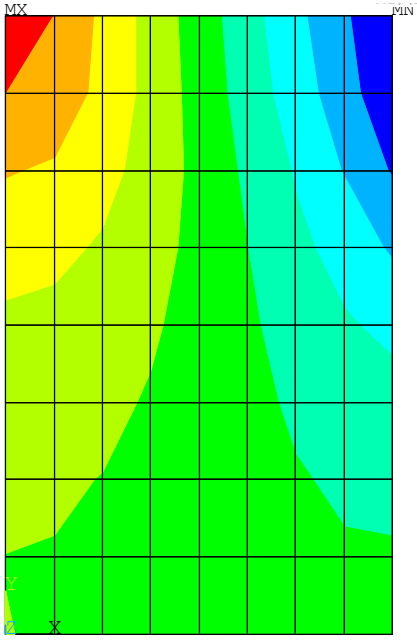
16



PLOT NO. 4
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.047092
SMN =-1.1916
SMX =59.8685

-1.1916
5.59286
12.3773
19.1618
25.9462
32.7307
39.5152
46.2996
53.0841
59.8685

64

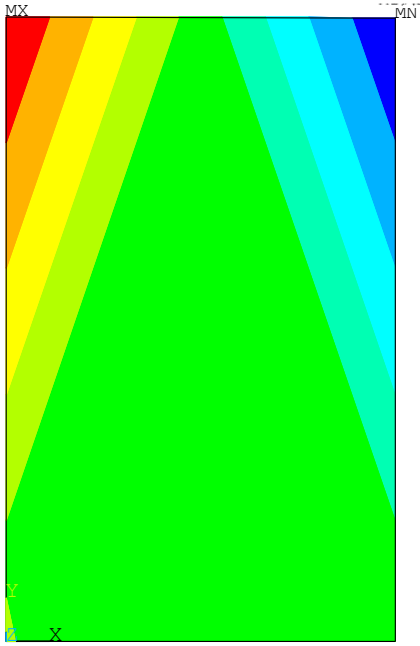


PLOT NO. 4
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.047175
SMN =-.352283
SMX =60.0386

-.352283
6.35781
13.0679
19.778
26.4881
33.1982
39.9083
46.6184
53.3285
60.0386

The impact of discretization on the quality of results *(8-node elements)*

1



PLOT NO. 6
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.04694
SMN =1.91405
SMX =58.086

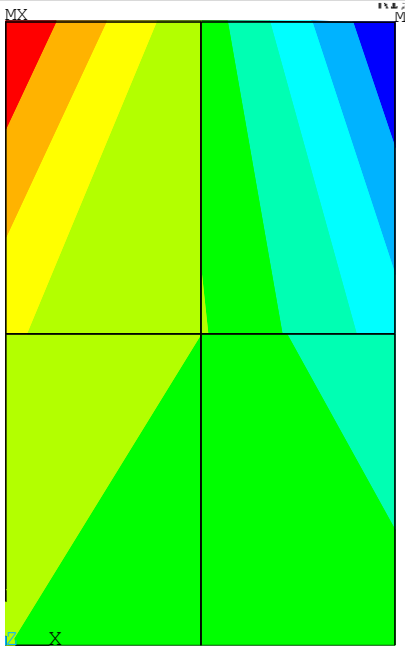
1.91405
8.15537
14.3967
20.638
26.8793
33.1207
39.362
45.6033
51.8446
58.086

σ_y

[MPa]

ELEMENT SOLUTION

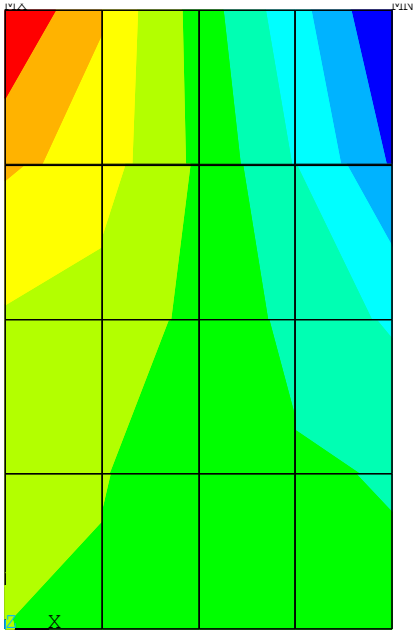
4



PLOT NO. 10
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.046767
SMN =-1.56578
SMX =58.9382

-1.56578
5.15688
11.8795
18.6022
25.3249
32.0475
38.7702
45.4928
52.2155
58.9382

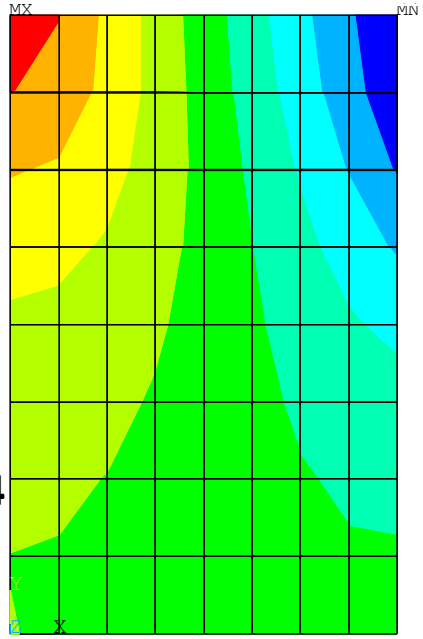
16



PLOT NO. 7
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.047092
SMN =-1.1916
SMX =59.8685

-1.1916
5.59286
12.3773
19.1618
25.9462
32.7307
39.5152
46.2996
53.0841
59.8685

64

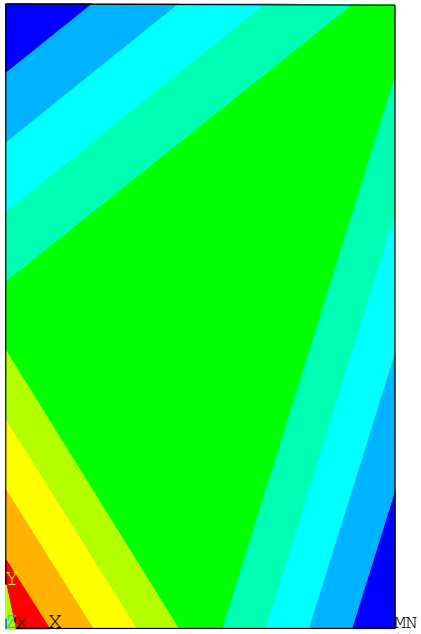


PLOT NO. 7
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.047175
SMN =-.352283
SMX =60.0386

-.352283
6.35781
13.0679
19.778
26.4881
33.1982
39.9083
46.6184
53.3285
60.0386

The impact of discretization on the quality of results *(8-node elements)*

1



PLOT NO. 7
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.04694
SMN =-6.81074
SMX =2.7595

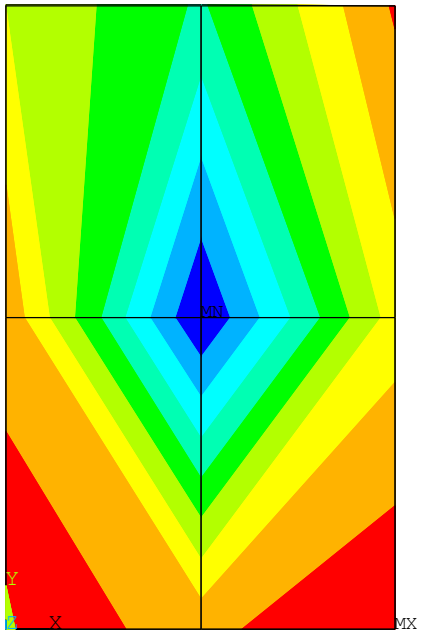
-6.81074
-5.74738
-4.68402
-3.62066
-2.5573
-1.49394
-.430576
.632784
1.69614
2.7595

τ_{xy}

[MPa]

NODAL SOLUTION

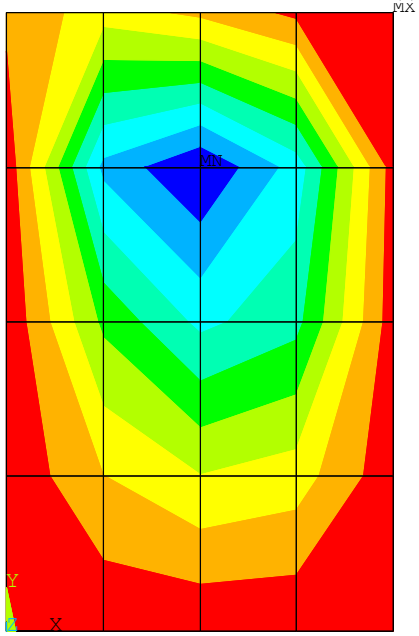
4



PLOT NO. 5
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SXY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.046767
SMN =-7.53406
SMX =.668435

-7.53406
-6.62267
-5.71128
-4.79989
-3.88851
-2.97712
-2.06573
-1.15434
-.242953
.668435

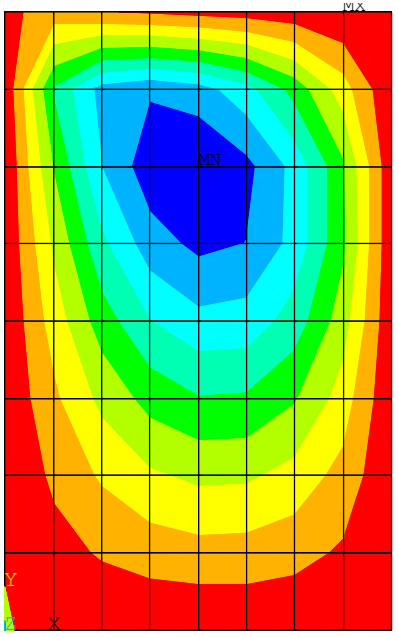
16



PLOT NO. 5
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SXY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.047092
SMN =-8.18541
SMX =.152147

-8.18541
-7.25902
-6.33262
-5.40623
-4.47983
-3.55344
-2.62704
-1.70064
-.774249
.152147

64

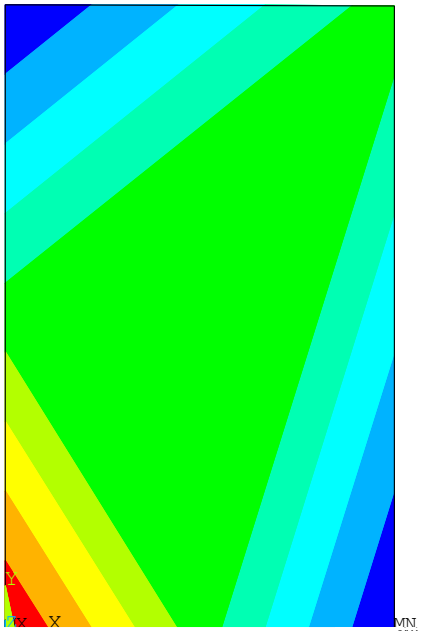


PLOT NO. 5
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SXY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.047175
SMN =-7.2591
SMX =.095601

-7.2591
-6.44191
-5.62472
-4.80753
-3.99034
-3.17315
-2.35596
-1.53878
-.721588
.095601

The impact of discretization on the quality of results *(8-node elements)*

1



PLOT NO. 7
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.04694
SMN =-6.81074
SMX =2.7595

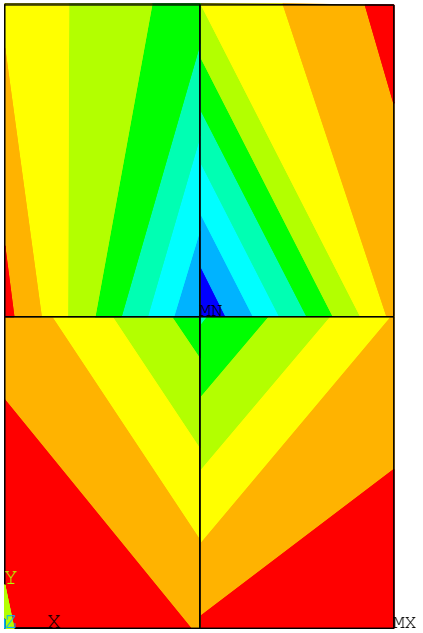
-6.81074
-5.74738
-4.68402
-3.62066
-2.5573
-1.49394
-.430576
.632784
1.69614
2.7595

τ_{xy}

[MPa]

ELEMENT SOLUTION

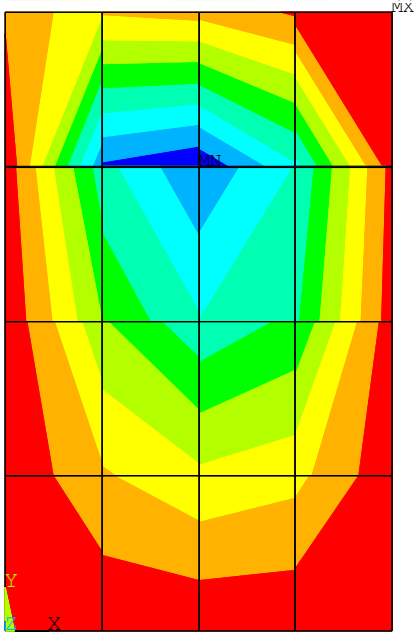
4



PLOT NO. 11
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.046767
SMN =-10.4579
SMX =.668435

-10.4579
-9.22163
-7.98537
-6.74912
-5.51286
-4.2766
-3.04034
-1.80408
-.567824
.668435

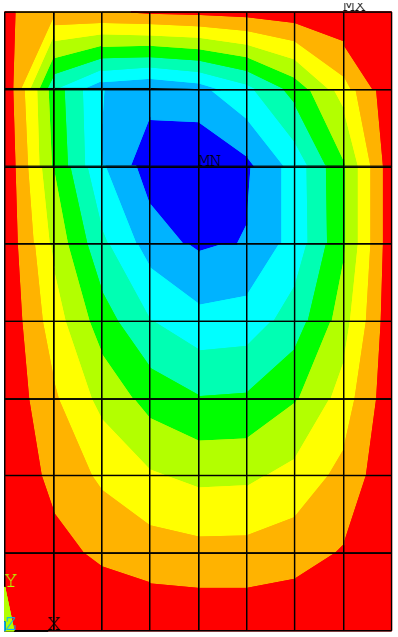
16



PLOT NO. 8
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.047092
SMN =-8.78477
SMX =.152147

-8.78477
-7.79178
-6.79879
-5.8058
-4.81281
-3.81982
-2.82683
-1.83383
-.840844
.152147

64



PLOT NO. 8
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.047175
SMN =-7.33453
SMX =.163931

-7.33453
-6.50137
-5.66821
-4.83505
-4.00188
-3.16872
-2.33556
-1.50239
-.669232
.163931